Student Achievement Division

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WHAT WORKS?
Research into Practice

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Research Monograph # 42

Are we placing too much emphasis on what children cannot do and underestimating their mathematical potential?

Trigonometry in Grade 3?

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"It's amazing that they're learning this math in Grade 3. I thought she couldn't do it but she really did. I hope you give more homework like this."
– Grade 3 parent

Research Tells Us

- Piaget's stages of cognitive development may misrepresent children's potential.
- If given opportunity, children can engage with math ideas beyond their grade level.
- Creating rich mathematical context enables teachers to cover curriculum content and offer children opportunities to engage with big math ideas.
- Strategies for creating rich mathematical context include using children's literature, arts-based communication, "low floor, high ceiling" approaches and hands-on exploration.

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Elementary school teachers work hard to cover grade-specific math curriculum expectations, but what if this is not enough? Ginsburg[1] suggests that "children possess greater competence and interest in mathematics than we ordinarily recognize" and that they should be challenged to understand big mathematical ideas and to "achieve the fulfilment and enjoyment of their intellectual interest" (p. 7). This position is supported by Moss[2] and her colleagues in their work with functions in Grade 4. By developing a stimulating, mathematically rich *context* for the *content* that students have to learn, teachers can address grade-specific curriculum expectations while offering students the pleasure of mathematical surprise. Young students, these researchers have shown, benefit from opportunities for using imagination and sensing mathematical beauty. This monograph shares our research in this area, highlighting the ways we have engaged children with ideas that are well beyond their grade level.

Underestimating Children

What if Piaget (or our interpretation of his work) is wrong?

Piaget's influence is pervasive. Egan[3] says that "development" in education is discussed and taught "almost exclusively in Piagetian terms" (p. 105). Piaget's constructivism, for example, has made a tremendous contribution to mathematics education by showing that "learning originates from inside the child"[4] (p. 260). But what if Piaget's other work, on stages of cognitive development, misrepresents children's potential?[3] Piaget[5] himself cautions about how generally his stages of development might apply: "We used rather specific types of experimental situations ... it is possible to question whether these situations are, fundamentally, very general and therefore applicable to any school or professional environment" (p. 46).

Some Helpful Websites

Performing Research Ideas

This site offers detailed lesson plans and videos from classrooms in which students explore big math ideas from higher grades. www.researchideas.ca

Windows into Elementary Mathematics

This site features interviews with Canadian mathematicians about elementary math topics. www.fields.utoronto.ca/mathwindows

Math + Science Peformance Festival

This site shares and celebrates students' artistic performances of mathematics, through songs, skits, and artwork. www.mathfest.ca

About Trigonometry

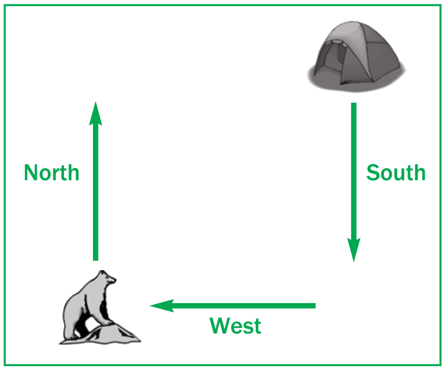
Trigonometric functions are used to model periodic phenomena such as sound and light waves, sunlight intensity and day length, seasonal temperatures, the changes in blood pressure as a heart pumps and relaxes and the weights of some wild animals over a year.

Papert[6] challenges the linear progression of Piaget's developmental stages and suggests the sequence is not in children's minds but in the "poverty of the culture" of schooling (p. 7). Fernandez-Armesto[7] adds that "generations of school children, deprived of challenging tasks because Piaget said they were incapable of them, bear the evidence of his impact" (p. 18). It is important to note that challenging Piaget's stages of cognitive development does not mean that we are necessarily challenging the organization of schooling around grades.

Is our world flat?

For example, we all learned in school that parallel lines are "straight lines that never meet," despite the fact that we live on a sphere where what is straight and what is parallel are mathematically much more complex and much more interesting. If parallel lines never meet, how could we solve this riddle?

Molly steps out of her tent. She walks South 1 km. She walks West 1 km. She sees a bear. She runs North 1 km, arriving back at her tent. How is this possible, and what colour was the bear?



A deeper understanding of parallel lines of longitude, however, leads us to the answer that the bear must be a polar bear and is, therefore, white, as Molly's tent must be at the North Pole.

Can young children think abstractly?

The principal at one of our K–8 project schools once announced, "This week's virtue is justice." If young students can learn about complex concepts such as virtue and justice, then certainly they are ready to learn that "parallel" lines can meet. Egan[8] notes that without abstract thinking children would not be able to develop language: "My point is that the development of language inevitably involves the use of abstractions and that abstract thinking – in the every day, rather vague sense of the word – is no less common in young children than is concrete thinking" (p. 47). As Ginsburg[1] suggests, although mathematics is "big," children's minds are bigger.

OVERVIEW OF OUR APPROACH

Working primarily in Grades 1–4 classrooms, our research explores how to engage young children with big math ideas from higher grades, such as trigonometry, limit and infinity, linear functions, optimization and non-Euclidean geometry. Along with children's literature and a focus on arts-based communication, we use these ideas to create a mathematically rich context for learning content. We use the following components to engage young students with the "abstract" ideas related to trigonometry while addressing curriculum expectations in data management (plotting bar graphs, interpreting data) and algebra (predicting, justifying, identifying, extending, and comparing patterns).

- **Children's literature** We use children's literature to engage children both cognitively and emotionally with math ideas.[9,10] Math in a story is easier to remember and easier to share with others. It is also easier for a teacher to implement, as a story contains a logical sequence (a lesson template) of ideas to explore and discuss.
- **Low floor, high ceiling** We design activities that have a low mathematical floor, allowing engagement with minimal math knowledge, and a high mathematical ceiling, so that concepts can be extended to more complex relationships and more varied representations.[2]

Math Waves

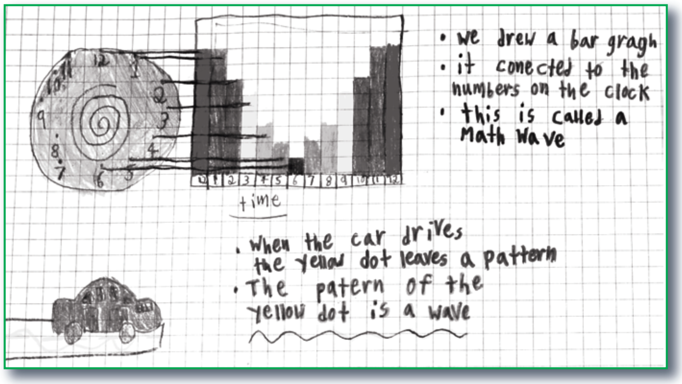
Students explore periodic phenomena, such as the path of a yellow dot on a car tire, as the car moves forward, and the height of the tip of the hour-hand on a clock, as time passes. This creates a rich mathematical context for students applying and developing skills in representing and analyzing data, and in predicting, testing and extending patterns.

What path will the yellow dot follow?

- Pose this problem to students: "Imagine a yellow dot on a car tire, turning as the car moves forward. What path will the yellow dot travel?"
- Model this, substituting a cylindrical cracker container for the tire, with a marker (representing the yellow dot) taped inside the rim, so the felt tip protrudes.
- Have pairs of students use chart paper to record their predictions, share their ideas and explain their thinking.
- Encourage students to test their predictions by rolling their own cylinders against a sheet of paper taped against a wall.
- Compare and discuss students' predictions (most often spiral paths) with the "wave" pattern that emerges.
- Repeat the process of predicting, explaining and testing with square-topped cracker containers.

Hickory, dickory, dock, the mouse ran up the clock …

- Read from the story Math Waves,[15] in which a mouse nibbles pieces of cheese placed at each numeral of a grandfather clock.
- As the mouse runs up and nibbles the cheese at one o'clock, students measure and plot its height. Ask students to repeat the exercise for the other hours on the clock, thus creating their own bar graphs.
- Discuss the wave pattern that emerges, and encourage students to share their ideas in a whole class setting. Discuss connections between the clock and the car activities. As one student noted, "The clock is a wheel … the arrows (hands) are … going around in a circle … so the clock is moving."



- **Mathematical surprise** We structure activities to increase the potential for students to experience "aha!" moments and the pleasure of mathematical surprise and insight.[11]
- **Hands-on exploration** We give students ample hands-on opportunities to explore concepts using multiple representations.
- **Arts-based communication** We use drama, art and music to help students develop effective communication skills and learn to relate good math stories.[12,13,14]
- **Home connection** Students share their learning with parents and we ask parents to comment on what their children learned and what they themselves learned.
- **Collective knowledge** Using student statements from their discussions and writing, we create concise summaries of our collective experience and learning. Students use these as a basis for skits or comics that summarize their learning. We also use them, along with feedback from parents, to create lyrics for class songs.
- **Audience** We create opportunities for students to share their learning with other classes, with parents, and with the wider world though the Math Performance Festival (www.mathfest.ca).

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Math waves all around us ...

- Discuss other phenomena that might create wave patterns.
- Have students plot bar graphs of sunrise times and average monthly temperatures and once again encounter wave patterns.

Making a home connection ...

- Have students use words, math symbols, diagrams and pictures to record what they learned and how they felt.
- Combine these to create a summary of their collective learning.
- Ask students use ideas from this summary to create comics on the theme of "What did you learn in math today?"
- Encourage students to take their work home and share their learning with parents.
- Ask parents to return feedback of what their child shared and what they themselves learned.
- Use this feedback to create lyrics for a song.
- Record students singing the song and share publicly at the Math Performance Festival (www.mathfest.ca).

In Sum

In math-for-parents workshops, I have asked parents how children reply when asked "What did you do in math today?" The common responses are "Nothing" and "I don't know." For this to change, students need two things. First, they need math that is worth talking about, with connections to big ideas that capture students' intellectual attention and engage their imagination. Using big ideas from higher grades has the advantage of helping students (and teachers) experience the grand narratives in school mathematics. This provides enrichment for all students and an early foundation for future success with complex mathematics concepts. Second, students need to develop arts-based communication skills for telling good math stories — stories that offer a sense of wonder and surprise about beautiful mathematics ideas. To meet these goals, our guiding question for lesson planning is, "Can we imagine our students relating to family and friends an engaging story about what they did in math today?"

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